Éva Bonifert Szabóné

Opportunities for Simplification in the Personal Income Tax Systems of the Visegrad Countries

Summary: Due to the numerous factors that can influence the impact of the tax system and redistribution, there is no single correct answer to the question of which composition of economic policy instruments needs to be applied to achieve a desired redistributive effect. The general aim of the study is to investigate in relation to the quantifiable parameters of income tax systems, whether the consideration of the aspects of fairness and justice does have an excessively negative effect on the simplicity of tax systems. The study investigates the possibilities of simplifying the personal income tax system's composition in some Central and Eastern European countries, while tax burden curves of the system remain as constant as possible. To this end, the study sets up a theoretical, simplified tax model, the parameters of which are determined by a computer program, in order to generate tax burden curves corresponding most closely to the curves of the real tax system. Based on the analysis, it can be established that the theoretical system – in some cases with restrictions – provides a good approximation to the tax burden curves of the investigated countries. The chosen simple model has a good degree of approximation to a real system that does not have significant breakpoints in its tax burden curves, nor does it use a taxation method that fundamentally modifies the system (e.g., splitting). Practical examples help to understand that a complex personal income tax system in a given country is not necessarily the only possible solution to achieve a given tax burden curve, the function may be reproduced with a good approximation constructed from simpler basic elements.

Keywords: optimal taxation, tax burden curve, personal tax system, simulation systems
JEL codes: H21, C61, K34
DOI: https://doi.org/10.35551/PFQ_2020_4_6

The state has a number of tools to redistribute income: the system of public expenditure is one option but the tax system is another very important pillar.

E-mail address: eva.bonifert@kormanyiroda.gov.hu

INCOME REDISTRIBUTION AS A PUBLIC FINANCE POLICY

The tax system itself may affect redistribution in many ways, through the regressivity, linearity or progressivity of the tax types. Income tax systems are often progressive, but

tax credits can significantly modify the degree of progressivity. Tax reliefs can be itemised themselves, any may also vary depending on certain factors, and the phase of the tax assessment and at the proportion of income to which the reliefs are applied may also affect the impacts of the tax system.

The principles of taxation are certainly best known from the works by Adam Smith (1940), Stiglitz (2000), and Musgrave and Musgrave (1984). In addition to the principles, the tension between the fairness and efficiency of the tax system is also an important topic of discussion in the literature relating to taxation, one of the most obvious areas of which is the defined number and degree of tax rates.

THE SIMPLE, OPTIMAL TAX SYSTEM

Due to the subjective and objective factors determining the impact of the tax system and the entire redistribution, there is no clear and single, perfectly correct answer to the question of which composition of economic policy instruments should be used in an economy to achieve the desired redistribution effect. However, one important aspect of the choice is clear: fairness versus efficiency and simplicity. The study seeks to contribute to the examination of this issue by analysing it.

Some key principles can therefore be defined to achieve a theoretical, simple, optimal and effective tax system. According to *Parragh* and *Palotai* (2018), an effective tax system can generate budget revenues by causing the least possible distortion in the allocation of economic resources and promoting economic growth where possible. According to *Bánfi* (2011), if a tax system is simple, there are no tax or tax base benefits in it, and there is no progressive taxation either, which he considers one of the types of tax benefits. Others argue that some form of progressivity considered to be equitable can also be achieved by combining a constant tax rate and significant tax benefits (linear taxation). *Giday* (2017) points out that progressivity is not equally effective for all tax types: he argues that, in the case of value added tax, a multi-rate system is more efficient and in the case of personal income tax, a flat-rate system can achieve the taxation objectives tax function properly.

However, even though the introduction of a perfectly simple tax system would make control easier and would encourage law-abiding behaviour, such a system would not be feasible in an open international economic environment. On the other hand, the tax system is an important area of regulation for governments because it is an obvious field for income redistribution. According to *Kürthy* (2010), a society with an extensive hidden economy, income secrecy and tax fraud also hinders the drastic simplification of the tax system from one moment to the next.

Increased progressivity of the tax system may, in some views, enhance fairness, yet has a deterrent effect on income-generating activities. Naturally, what is considered to be the most appropriate tax rate also depends on perceptions of fairness and the ability of taxes to hold people back from working (Heady, 1993).

What shapes the degree of optimal progressivity of the tax and transfer system? - ask *Heathcote, Storesletten and Violante (2017).* On the one hand, a progressive tax system can offset inequalities in the basic conditions of taxpayers and replace gaps in individuals' income risk sharing in society. On the other hand, progressivity reduces incentives to work and invest in future skills and exacerbates the externalities of valuable public spending. Heathcote, Storesletten

and Violante (2017) have developed an equilibrium model that explores these compromises. They argue that investment in skills, flexible labour supply, and externalities associated with valuable government procurement play a similar role in limiting optimal (or desired) progressivity.

A number of authors have examined the optimal factors of income taxation, in particular *Mirrlees* (1971), who argues that the optimal tax table is close to a linear one. *Mankiw et al* (2009) came to a similar result. In contrast, some found that marginal tax rates should be 'U-shaped': higher tax rates should be applied for low and high income than in the middle of the income distribution (e.g., Diamond, 1998; Saez, 2001). *Heathcote and Tsujiyama* (2019) believed that the optimal tax table depends on the pressure to increase public revenues: as financial pressure increases, the curve of tax rates of the optimal tax table changes from 'flat' to a U-shape.

A particularly interesting model for optimal taxation of income has been developed by Heathcote, Storesletten, and Violante (2019). The model examined the optimal taxation of incomes and the degree of tax progressivity as a function of age. The overlap-generation model includes investment in the acquisition of skills, flexible labour supply, diversity of sacrifice due to the cost of work and skills acquisition, wage risk, and life-cycle productivity profile. They found that the progressivity of income taxation should be U-shaped, while the average marginal tax rate should be increasing and concave as a function of age.

However, we can also understand that the more efforts are made to combine the aspects of fairness and efficiency in a tax system, the more complicated it will be: a multi-factor, progressive tax system becomes too complex due to rules designed to compensate for its side effects (Balogh 2013).

OPTIONS FOR SIMPLIFYING THE TAX SYSTEMS

In connection with the simplification of tax systems and with the recently observed reduction of administrative burdens, it should not be forgotten that optimal taxation may also be approached by simplifying tax rates and tax rates. Moreover, the simplification of the way the tax is calculated can itself lead to a reduction in administrative burdens.

As simple as this statement may sound, we must consider it important that not only the taxation instruments used but also the setting of the parameters of the instruments are also decisive in the assessment of a tax system (how fair and how efficient and simple it is). In order to achieve a targeted effect (such as giving preference to lower income earners, wage earners, etc.), it is not enough to choose a specific tax instrument that is generally used for this purpose, setting its parameters is essential because the mathematical correlations of the parameters also strongly influence the shape of the tax burden curve. In connection with the tax elements preferred by different theoretical approaches, their mathematical substitutability may also arise. Thus, according to our first hypothesis, a given tax burden curve can also be defined using multiple basic taxation elements.

The simplest method to prove the hypothesis starts with a simple, flat-rate system containing also a tax-free bracket. The tax impact of the tax-free band can in some cases be determined with a tax credit so that the tax burden curve does not change at all or only very little, so the two basic elements can even replace each other. The differences between the applicability features of the tax credit and the tax-free bracket, through to be fundamental, are true only if certain conditions are met.[1] By adjusting the tax burden to the tax burden of the income tax system using a tax-free

bracket, the parameters of the tax credit can be defined so that it would result in exactly or almost exactly the same tax burden on taxable persons as a system using the tax-free band. It should also be pointed out in this context that a completely simple, seemingly flat-rate income tax system does not necessarily mean that there is indeed a single tax rate and no progressive elements.[2]

Certain elements of personal income tax systems are introduced in order to achieve fairness as precisely as possible, but they complicate the tax system itself. Although these elements might achieve the desired effect, they still cannot be applied due to the excessive complexity of the system, and even the perfect achievement of the desired effect may be questionable due to the difficulties of application and the opportunities of tax evasion. According to our second hypothesis, there is a specific system of personal income tax applied in practice, where the expected effect on the tax burden can be achieved with simpler means or may be approximated with good results.

The further part of the study examines the possibilities of simplifying the parameters of the personal income tax system, i.e. the tax rates, benefits and structure of certain tax systems within a certain framework along these ideas. In this context, however, it uses new aspects: it examines the potential simplification rom the point of view of the tax burden on taxpayers and not from the point of view of revenue (considering it being unchanged as an objective).

Of course, the idea arises that the analysis should also include the analysis of the Hungarian system and the possibilities of simplification according to these aspects. It is important to state that the aim of this paper is not to provide theoretical guidance for the development of a new national personal income tax system, but it intends to substantiate the raison d'être of a simple tax system with a new approach with the factors of a complicated tax system that takes into account all aspects of fairness. Considering that in the Hungarian system personal income taxation is generally simply defined in terms of tax rates, tax brackets and tax rates, we do not see any further simplification option based on the method outlined below. In the rest of the paper, we examine the generally more complicated personal income tax system of the other Visegrad countries than in Hungary. The analysis of the development of the Hungarian system according to the described aspects may be worth examining in a longer period of time, during which the possible missed simplification options of the previous systems can be examined in the framework of another study.

The analysis in this paper can help, as examined also by *Balogh* (2013), to understand the complex relations between the justice (fairness) and efficiency of the tax system, and the related choice of values.

On the other hand, when making a planned decision that also affects the tax burden, decision-makers often aim to keep the tax burden of a group of taxpayers as constant as possible, despite some change in the system. The newly set up calculation system can help to set up the parameters of the personal income tax system for a pre-determined tax burden curve. Defining freely, in advance, the number of tax rates of the income tax system, the types of possible benefits and credits, the computer program developed by the author is able to calculate the associated tax system parameters that best fit a particular tax burden curve.

THE RESEARCH METHOD AND THE ATA USED

In order to determine whether the tax burden on the income tax systems of the countries examined can be approximated in some way with other tax elements or possibly with

simpler methods, and whether the same tax impact can be achieved, it is necessary to define the basic elements of a theoretical tax system that typically meets the criteria of a simple tax system, i.e. to set up a simple, own tax model of its own (the pre-selected elements of the model will be described later).

We are interested in whether the theoretical tax model can be parameterised to approximate the tax burden curves of the studied countries as expected. If so, or if the solution succeeds only in part, the question arises whether complex tax systems should be maintained at all costs, possibly in order to achieve the most equitable income tax system possible. Are there any, and if so, what are the elements that, at least solely in terms of their effect on the shape of the tax burden curve, are not worth maintaining, because the tax system can impose the same tax burden without them?

The parameters of the theoretical tax model approximated to the real tax system can be calculated in a special program system called MATLAB, developed for performing numerical calculations, with the help of a program[3] written by the author in order to obtain the most accurate tax burden curve. The theoretical model is a single-bracket (flat-rate) personal income tax system with an itemised tax credit, thus a flat-rate tax system in the general sense, where an itemised tax credit for children and spouses can also be claimed, where a negative tax can also be used in respect of the tax allowance for children.

The parameters of the theoretical tax model to be determined by the program are the following:

- the tax rate (percentage),
- the rates of itemised tax credit (HUF),
- the tax allowance for children (HUF) and
- the tax allowance for the spouse (HUF).

The program searches for the parameters using the built-in *fmincon* optimisation algorithm with the method of limited nonlinear optimisation (non-linear programming) based on the minimum of the residual sum of squares (sum of the squares of the differences between the theoretical tax burden calculated for the real income levels of the given country on the basis of the parameters searched and the real tax burden in the OECD data) defined in the program.

The data forming the basis of the comparison are taken from the OECD online taxation database[4], which includes data series on central personal income tax rates, calculated on the basis of gross wages as a percentage of average earnings for different countries and family types. In terms of family type, in the OECD database contain only data on the taxation of

- single people with no children,
- single people with two children,
- single-earner, married couples with no children,
- single-earner, married couples with two children,

therefore, the analysis cannot undertake a detailed examination by household type. For this reason, we present possible simplification options for income tax systems with data for these four selected family types. The analysis was based on data of 2018. The OECD tax data refer to income levels corresponding to 50–250 percent of the average wage, so these income thresholds also form the income constraints of our analysis.

Considering that the four family types are taxed in the same tax system (only the availability of the child tax allowance and the spouse tax allowance are different between them in our theoretical tax system), the program searches for the optimal tax parameters based on the combined error function of the four family types.

The program was run with several initial values of the searched parameters in order to prevent it from 'sticking' to a local optimum situation when searching for the minimum of

the function, and that it can find the global optimum if possible. The process of curve fitting is shown in *Figure 1*.

In the performed model calculations we had to use simplifications and restrictions, partly due to the scarcity of available data and partly due to the practical feasibility of modelling.

The restrictions on household types and income levels have already been described. With regard to the calculations, it should also be noted that the real tax systems underlying the analysis have also been simplified: tax parameters (e.g., allowances) are used, which are as general as possible, least specific and applicable to most taxpayers. Hence only the official tax table (the tax rates), some tax reliefs relating to employment and tax base reductions and tax allowances that can be used in connection with the support of families and married couples, can be taken into account. However, these simplifications are also used in the OECD database: the tax burden data relating to the real system published by the OECD are also based on the elements of *Table 1*, and the breakdown of the data by family type also facilitates the application of these allowances in the model.

However, while examining real income tax systems, we also considered two allowances - given that they are also referenced in the OECD database - for which there is no separate data set (family model) in the OECD database for the group of taxpayers actually using them. Similarly to the OECD data set, however, we are forced to assume that every child of a taxpayer with children are pre-schoolers (this will be relevant later) and every earning taxpayer has one job.[5]

In modelling, we also assume that social and health contributions are mandatory for everyone, and only wage income is taken into account.

If we had data series for more than one household type, we could perform a complex analysis in which the changes and possible simplification of income tax systems could be extended to a broader range, taking into account its impacts. The lack of data narrows the possibilities of our calculations, but, as we can see below, the analysis still has room for manoeuvre. In this study, we seek transparency and wish to present simplification opportunities based on individual examples of the tax system.

RESULTS OF THE RESEARCH

In the study, we first compare the main characteristics of the personal income tax systems of the given countries, which are presented in Table 1. The table groups the elements of tax systems according to their subject, indicating their place in income taxation with colours. The colouring also shows whether a particular element of the tax system applies to all income categories or not.

Elements of the theoretical model

The theoretical model is a single-bracket (flat-rate) personal income tax system with an itemised tax credit, thus a flat-rate tax system in the general sense. The system also allows for itemised tax allowance for children and spouses. Similarly to the systems of the studied countries, the model also allows for the use of negative tax for the tax allowance related to children. The specific parameters of its elements are determined by the program as previously described.

Czech Republic

With regard to the special solidarity surcharge, the tax credit and other benefits, a progressive, multifactorial system in terms of allowances

Figure 1

METHOD OF CURVE FITTING BASED ON THE THEORETICAL MODEL

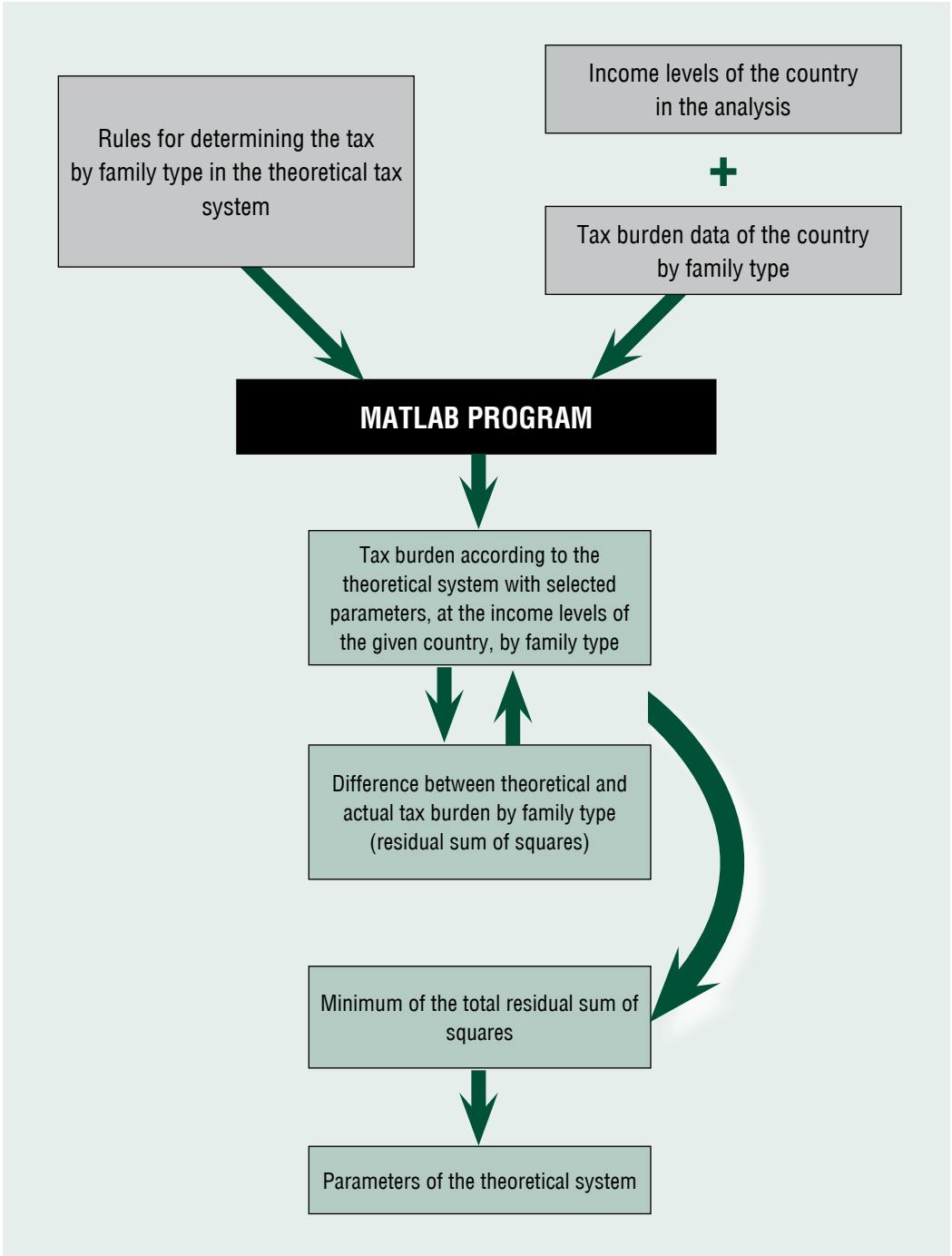

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Table 1

MAIN CHARACTERISTICS OF THE PERSONAL INCOME TAX SYSTEMS OF THE EXAMINED COUNTRIES IN 2018, COMPARED TO THE PARAMETERS OF THE THEORETICAL MODEL

	Czech Republic	Slovakia	Poland	Theoretical model
Special method for determining the tax base	Super gross	–	(See allowance related to the spouse)	–
Allowance of social security and health insurance contributions	–	Allowance of social security and health insurance contribution (percentage)	Allowance of the contributions of the social system (percentage)	–
			Tax credit of the health insurance contribution (percentage)	
Allowance for work expenses	–	–	Allowance for work-related expenses (itemised)	–
Number of tax rates	Tax rate 1	Tax rate 1	Tax rate 1	Tax rate 1
	Speciel solidarity surcharge	Tax rate 2	Tax rate 2	–
Allowance related to the taxpayer	Tax credit (itemised)	Basec allowance (degressive, itemised)	Tax credit (degressive, itemised)	Tax credit (itemised)
Allowance related to the spouse	Tax credit related to the spouse (itemised)	Allowance related to family status (degressive, itemised)	Splitting	Tax allowance related to the spouse (itemised)
Tax allowance related to children	Tax credit for children (itemised, can be considered as a negative tax)	Tax credit for children (itemised, can be considered as a negative tax)	Tax credit for children (itemised, can be considered as a negative tax, ceases over a specified income)	Tax allowance related to children (itemised)
	Tax credit for pre-school children (itemised)	–	–	–

Note: Explanation of the colours used in the table: light green = special method of tax base calculation, light gray = tax base reducing factor, dark gray = tax rate, dark green = tax allowance, white = there is no such element in a given tax system, striped = regressive allowance or tax element (tax rate) outside the income categories of the OECD data series

Source: own edited

was used in the Czech Republic in 2018, and this may be considered as the simplest income tax system for the countries studied according to the comparison in Table 1.

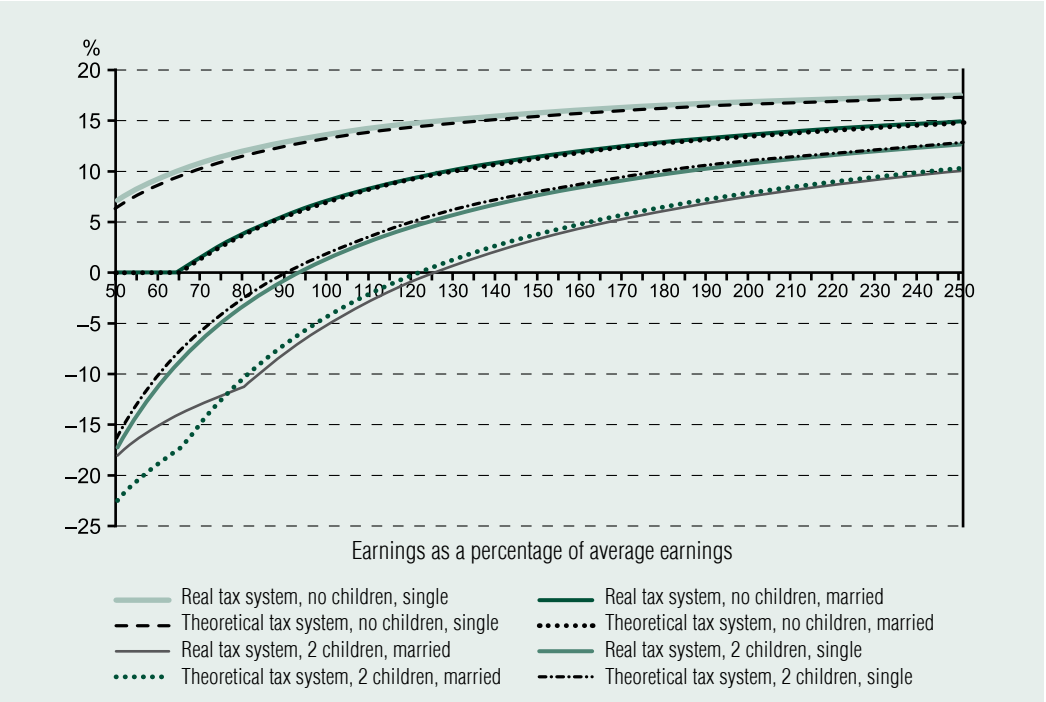
Thus among the tax systems of the examined countries, the theoretical model is the closest to the Czech system, considering that, in terms of the number of parameters of the tax systems, the theoretical tax model with a flat-rate, itemised tax credit is simpler than the Czech tax system only in terms of super-gross tax assessment and allowances related to children. (In our analysis, the Czech tax system is also considered as a flat rate system as the solidarity tax has to be paid on a high income that is no longer included in the OECD data series, so it cannot affect our analysis either.[6])

We calculate the parameters of the flat-rate theoretical system and examine the opportunity for simplification along the theoretical system based on the model defined in the MATLAB program system.

Figure 2 shows that the single-bracket theoretical tax system provides an almost perfect approximation to the Czech income tax system. The theoretical system does not strongly approximate the tax burden curve of the real income tax system only for the low-income categories of the married couples with two children. This discrepancy is due to the fact that the theoretical tax system includes only

Figure 2

FLAT-RATE TAX, THEORETICAL INCOME TAX SYSTEM AND TAX BURDEN OF THE CZECH CENTRAL INCOME TAX SYSTEM FOR DIFFERENT FAMILY TYPES, 2018



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one type of tax allowance for children, which can also be considered as a negative tax; in a real tax system, the tax allowance for children also works this way, however, the benefit for pre-school children cannot be used if the total of all the discounts that cannot be considered as negative taxes reaches the calculated tax. Thus, at low income levels, where child allowances could only be claimed as a negative tax because the amount of the calculated tax is reached by the other allowances, thus in the theoretical tax system, in total CZK 46,808 can be taken into account for two children, which is the total amount of the allowance for children. However, in a real tax system, it is less than CZK 34,608, which is equal to the amount of the basic tax allowance for children.

The parameters of the theoretical system would be the following based on the optimisation program run:

- tax rate: 20 per cent
- itemised tax credit: CZK 26,028 p.a.
- tax allowance for children: CZK 21,560.5 p.a./child
- tax allowance for married couples: CZK 23,955 p.a.

In assessing the extent to which the tax burden curve of the theoretical system fits the real tax burden curve, the degree of relative error can help us, as used in statistics.[7] To assess the success of the curve fitting, we therefore determined the magnitude of the relative error for all four family types, which is shown in *Table 2*. The table shows that the size of the relative error s slightly higher than 10 per cent only in the case of the single-earner married families with two children, which would still be a good match according to the widely accepted position in the literature. The difference is caused by the already described difference in the use of the allowance for pre-school children by single-earner married couples with two children.

The function approximation is shown in Figure 2, the slight differences for the other family types are also due to the fact that the allowance for pre-school children is taken into account: given that the system seeks the minimum of the total residual sum of squares, the break seen in the function of married couples with two children also affects the other three function approximations. If the difference due to the difference in the use of the allowance for pre-school children is taken into account with less weight when searching for the minimum of the total residual sum of squares,[8] then, with the exception of the already mentioned low income categories of the married couples

Table 2

THE MAGNITUDE OF THE RELATIVE ERROR FOR THE THEORETICAL TAX SYSTEM INTEGRATED INTO THE CZECH INCOME TAX SYSTEM

Family type	Magnitude of the relative error
Childless single	0.0246
Two children, single-earner married	0.1564
Childless, single-earner married	0.0141
Two children, single	0.0559

Source: own calculation

with two children, the tax burden curves of the theoretical system almost perfectly approximate the tax burden of the real system.

It follows that the tax burden curves of the Czech income tax system could also be determined with a good approximation by the elements of a somewhat simpler system in terms of the number of parameters of the tax system, for which the computer program can also calculate the specific parameters. Considering the invariance of the tax burden curves as an objective, the super-grossing used by the Czech system as a parameter in the tax system could have been omitted by applying the factors of the theoretical tax system. The allowance related to pre-school children (given that it is subject to different tax principles in the Czech system than the child pre-tax allowance) could be replaced in part, in most half of the income levels, if only the[9] perfectly matched tax burden curve is the goal.

Slovakia

The personal allowance and the spousal allowance were applied in Slovakia even before the major changes in 2013, in a system similar to the current one, which decreases in parallel with the increase in income, so an additional tax rate was added to a slightly more complicated flat-rate tax system in 2013.

Table 1 shows that the Slovak income tax system uses partially different tools than the Czech system: In the Czech Republic, the tax base is determined with the super-grossing method, while in Slovakia the amount paid by the employee as a contribution to the social and health insurance system is not included in the tax base. An important difference in principle is that in the Slovak system, itemised benefits (excluding the allowance for children) decrease with the increase in income, thus strengthening the fairness of the system, but also making the system more complicated. (However, the reduction in the marital allowance depending on the taxpayer's income occurs at such a high income that it is no longer included in the OECD data series.) In addition, the personal and marital status allowance can be considered as a tax base allowance in Slovakia, while similar items can be considered as a tax allowance in the Czech Republic.

Comparing the theoretical flat-rate tax model with an itemised tax credit and the Slovak income tax system, the theoretical tax system includes, in contrast with the Slovak system, the amount paid by the employee as a contribution to the social and health insurance system. In addition, in the Slovak system, the personal allowance can be taken into account degressively in parallel with the increase in income, but in the theoretical system such more complex elements have not been introduced. The second tax rate applied in the Slovak income tax system (and the decrease in the allowance related to the marital status) enters the system at a high income that is no longer included in the OECD data series, so it does not affect our analysis.

The question is therefore whether the tax burden curves of an income tax system operating with a gradually degressive rate of discount can be approximated by our theoretical tax system with fewer and only itemised allowance.

According to *Figure 3*, the tax burden curves of the flat-rate, theoretical system provide a good - almost perfect - approximation to the real tax burden data in the case of the Slovak system as well.

The parameters of the theoretical tax system would be as follows:

- tax rate: 17 per cent,
- itemised tax credit: EUR 783 p.a.,
- tax allowance related to children: CZK 258.5 p.a./child,
- itemised tax allowance: EUR 740 p.a.

Figure 3

FLAT-RATE TAX, THEORETICAL INCOME TAX SYSTEM AND TAX BURDEN IMPOSED BY THE SLOVAK CENTRAL INCOME TAX SYSTEM ON THE DIFFERENT FAMILY TYPES, 2018

Source: own edited

In this case, we can also calculate the magnitude of the relative error for each family type, which is summarised in *Table 3*, in order to assess the adequacy of the curve fitting. The table shows that this is very low for all family types, well within acceptable levels.

It can be assumed that the smaller difference between the tax burden curves of the Slovak and the theoretical tax system may be caused by the revocation of the personal allowance. In connection with this, the question may also be whether, if the personal allowance were not subject to revocation in the real system, our theoretical system would give a completely accurate approximation to the tax burden curves of the Slovak income tax system. To answer this question, we calculated the tax burden of the real Slovak system without the revocation of the personal allowance (for the sake of simplicity, the presented contribution benefit, which has an influence on the minimal part of the tax burden curve, has also been removed from the system), and for this we ran the approximation procedure programmed into MATLAB.

Thus, in this alternative curve fitting, the Slovak and theoretical systems (in terms of the number and type of parameters of the tax system) differ only in the tax base allowance on the amounts paid as contributions to the social and health insurance system.

Table 3

THE MAGNITUDE OF THE RELATIVE ERROR FOR THE THEORETICAL TAX SYSTEM INTEGRATED INTO THE SLOVAK INCOME TAX SYSTEM

Family type	Magnitude of the relative error
Childless single	0.0216
Two children, single-earner married	0.034
Childless, single-earner married	0.028
Two children, single	0.0279

Source: own calculation

If no revocation of the personal allowance were applied in the Slovak system, the tax burden curves of the theoretical system (leaving all other factors unchanged) would follow exactly the tax burden of the[10] modified Slovak system (with minimal relative error), thus, the smaller difference between the tax burden curves is most likely due to this withdrawal.

If we look at the difference between the real and the theoretical system from the aspect of the amount of tax paid - for the sake of simplicity only in the case of a single-earner family model with two children - the largest difference is at the highest income level, 250 per cent of average earnings (instead of a tax of EUR 3,317 p.a., EUIR 202 less would have to be paid in the theoretical system than payable actually; this difference equals 0.67 per cent of gross earnings). If no revocaction were applied to the personal allowance in the real system, the tax burden of the thus modified real system would be EUR 3,017 for 250 per cent of the average earnings. The difference between the tax burden of the modified real system and the original system is EUR 300 (naturally, the difference in tax payment between the modified real system and the approximate theoretical tax burden would be negligible), the tax burden of the theoretical model approximated to the original system is between the two, so the theoretical system can to some extent also replace revocation.

In summary, the tax burden curves of the Slovak system could be presented with the parameters of the theoretical system without the application of tax base allowances on the amounts paid as contributions to the social and health insurance system. The theoretical system can also partially replace the withdrawal of itemised allowances.

The obvious purpose of the tax base allowance of the social security and health insurance scheme is to exclude from income taxation the income which is the subject of another taxation scheme. However, the aim of this paper is not to evaluate this instrument - or the income taxation parameters of any other state under review - but merely to show whether it is possible to approximate or possibly achieve tax burden functions created according to specific principles using simplified parameters.

Poland

Poland has a complex progressive income tax system with a multi-rate, tax-free income component, and these items are accompanied

by additional allowance related to specific purposes.

The Polish income tax system also uses parameter elements typical of the region, but in a significantly modified form compared to the theoretical, simplified tax system according to the model. The difference between the theoretical income tax system and the Polish system is that the theoretical system uses one tax rate, while the Polish system applies two. In terms of the number of other parameters, the allowance on the contribution to the social and health insurance system, an allowance for work expenses and the revocation of the allowance for children are additional items in the Polish system. In contrast to the systems of other countries studied, the revocation of the child allowance does not take place in a gradually degressive system: above a certain income it simply ceases. Another important difference is that Polish taxation applies splitting, while the theoretical system applies the itemised allowance of spouses to lower-income spouses, in order to balance out taxation on family income.

According to the optimisation program, the tax burden curves of the single-rate theoretical system approximate the curves of the Polish system. However, our model cannot perfectly reproduce the break point in the tax burden curve for childless single persons due to the bracket limit or the break due to the termination of the child allowance above a certain income without revocation. *Figures 4 and 5* show the results.[11]

Figure 4

FLAT-RATE TAX, THEORETICAL INCOME TAX SYSTEM AND TAX BURDEN IMPOSED BY THE POLISH CENTRAL INCOME TAX SYSTEM ON CHILDLESS SINGLE PERSONS AND SINGLE-EARNER MARRIED COUPLES WITH TWO CHILDREN, 2018

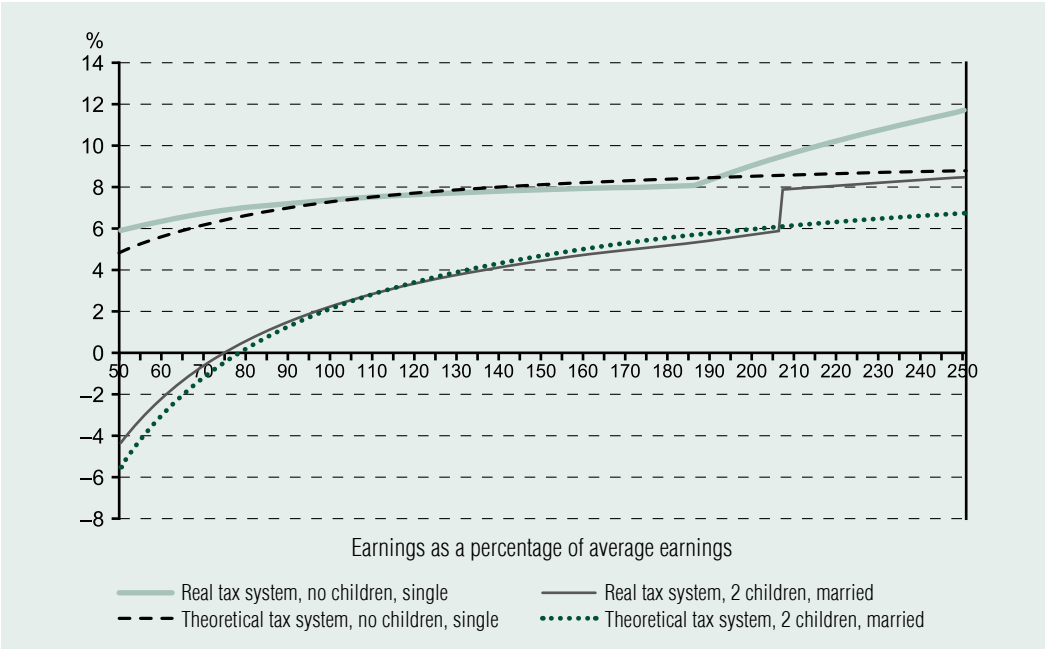

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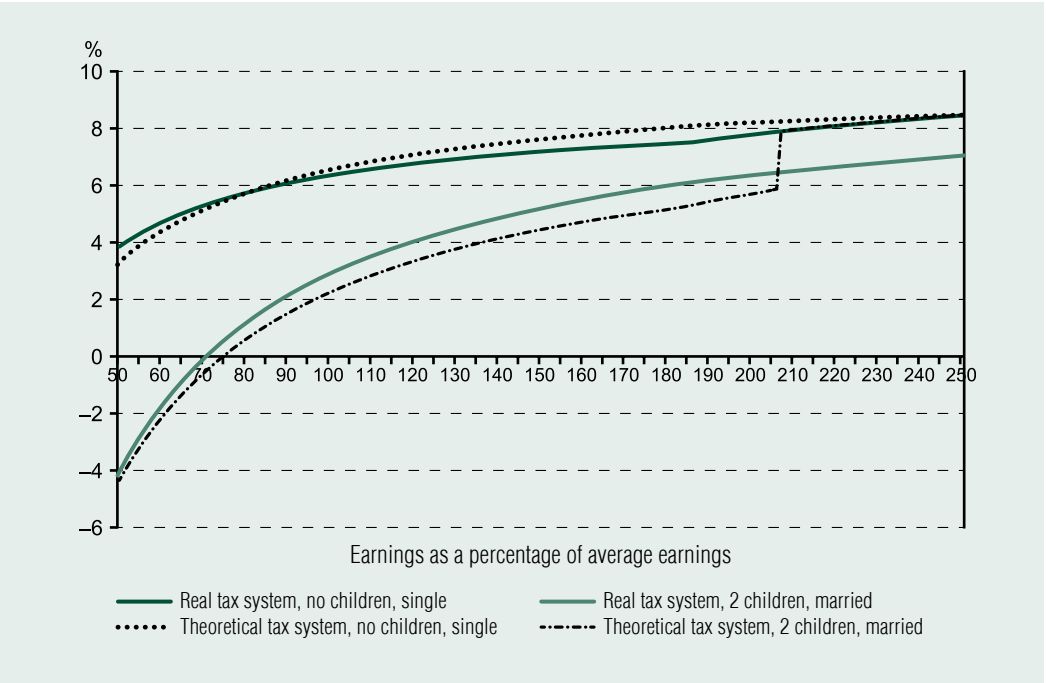
Figure 5

FLAT-RATE TAX, THEORETICAL INCOME TAX SYSTEM AND TAX BURDEN IMPOSED BY THE POLISH CENTRAL INCOME TAX SYSTEM ON CHILDLESS, SINGLE-EARNER MARRIED COUPLES AND SINGLES WITH TWO CHILDREN, 2018



Source: own edited

The parameters of the theoretical tax system would be as follows:

- tax rate: 9.8 per cent
- itemised tax credit: PLN 1,370 p.a.,
- tax allowance related to children: PLN 1,192.5 p.a./child
- tax allowance for married couples: PLN 396 p.a.

In the case of the Polish income tax system, the relative errors of curve fitting are summarised in *Table 4*. In this case, the relative error is only fully acceptable for the childless single-earner married family type, given that the tax burden curves of the Polish system full of breakpoints are only partially followed by the simplified theoretical system.

Although the cut-off point for the cessation of the child allowance and the tax burden curve for childless single persons due to the bracket limit (as splitting is used in families, which causes a different bracket limit breakpoint in the real system for families and children without other income levels) is not fully followed tax burden curve of the flat-rate theoretical system, the difference in income tax between the Polish and theoretical systems is up to 1-1.75 percent of income for childless spouses, single-earner couples with two children, and single couples with two children, and is up to 3 percent of income for single persons without children.

We can also examine the approximation of

Table 4

THE MAGNITUDE OF THE RELATIVE ERROR FOR THE THEORETICAL TAX SYSTEM INTEGRATED INTO THE POLISH INCOME TAX SYSTEM

Family type	Magnitude of the relative error
Childless single	0.1225
Two children, single-earner married	0.1741
Childless, single-earner married	0.0494
Two children, single	0.1725

Source: own calculation

the theoretical model if the immediate and complete cessation of the child tax allowance over a specified income can also be applied in the model. The graphs below show that even with this one, smaller preference, the model gives a much better approximation to the real system.

In this case, the optimisation program approximated the line of the tax burden curve according to the model to the tax burden curve of families, so the only major difference for childless single persons remained the approximation of the breakpoint due to the bracket limit. For the other family types, due to the use of splitting, the tax bases according to income levels according to the OECD data series are so low that the second tax bracket does not apply to them. For families, the approximation of the tax burden function is almost perfect, as shown in *Figures 6 and 7.*[12]

In this case, the parameters of a theoretical flat tax system would be as follows:

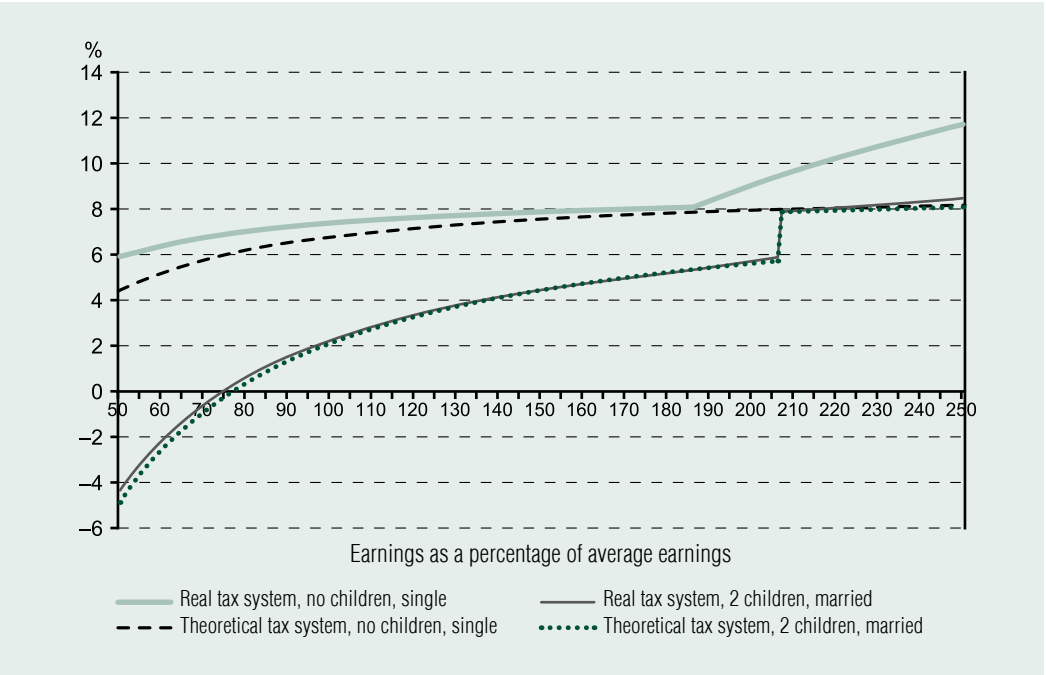
- tax rate: 9.1 per cent
- itemised tax credit: PLN 1,262 p.a.,
- tax allowance related to children: PLN 1,202 p.a./child, which ceases above the income of PLN 112,000,
- tax allowance for married couples: PLN 125 p.a.

The size of the relative error also shows that the accuracy of curve fitting in this case is already acceptable for most family types, and only exceeds 10 per cent (16.56 per cent) acceptable in the literature for childless single persons.

Based on the above therefore, in the case of the Polish scheme, without the factors of the allowance for the amount paid as a contribution to the social security and health insurance scheme, the allowance for work, expenses the withdrawal of the tax credit and using one instead of two tax rates, the model can approximate the tax burden of the Polish system but the theoretical model approximates the real tax burden curves really well if the theoretical system also applied the cessation of the tax allowance related to children above a certain income, or perhaps splitting. (By way of comparison, it could also be examined, as part of another study, how large difference in the tax burden would result from the removal of an allowance element from the real system, compared to the approximation of our model but, due to lack of space we could not undertake this task now.)

Figure 6

FLAT-RATE TAX, THEORETICAL INCOME TAX SYSTEM AND TAX BURDEN IMPOSED BY THE POLISH CENTRAL INCOME TAX SYSTEM ON CHILDLESS SINGLE PERSONS AND SINGLE-EARNER MARRIED COUPLES WITH TWO CHILDREN, 2018



Source: own edited

SUMMARY

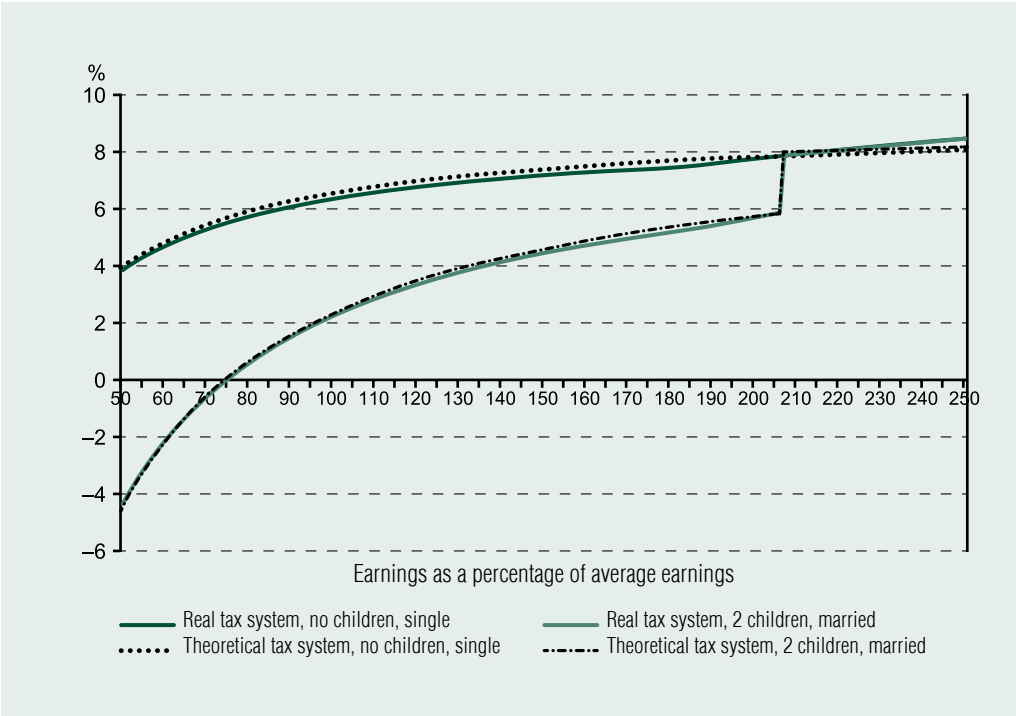
It is clear from the comparison of the parameters used in personal income taxation of the examined countries that the main basic elements of the benefits and allowances are very similar in each tax system (e.g. personal allowance, allowance for children, allowance for spouses). Basically, the potential multiple application of the main parameters means the differences in the income tax systems of the given countries (for example, whether revocation is applied for income higher in the case of a certain allowance element, whether a particular item is percentage or itemised, how is the taxation of spouses preferences).

The width of the income rage to which a taxation element is applied is an important feature of taxation elements. In several cases, it was common for the countries observed to use certain elements for such a narrow income group (for example, the Czech solidarity tax and the Slovak second tax rate only apply to a particularly high level of income), that they are outside the OECD's relatively broad income band data, so we could not even examine their effects.

In the course of the study, we reviewed the systems of countries that use increasingly complex tax elements. Summarising the results, the theoretical, simpler tax model could replace several factors of the examined tax systems (allowance affecting the tax base

Figure 7

FLAT-RATE TAX, THEORETICAL INCOME TAX SYSTEM AND TAX BURDEN IMPOSED BY THE POLISH CENTRAL INCOME TAX SYSTEM ON CHILDLESS SINGLE-EARNER MARRIED COUPLES AND SINGLE PERSONS WITH TWO CHILDREN, 2018



Source: own edited

Table 5

THE SIZE OF THE RELATIVE ERROR IN THE CASE OF THE THEORETICAL TAX SYSTEM INTEGRATED INTO THE POLISH INCOME TAX SYSTEM - IN THE CASE OF CHILD ALLOWANCE WHICH CEASES AT A CERTAIN INCOME LEVEL

Family type	Magnitude of the relative error
Childless single	0.1656
Two children, single-earner married	0.0387
Childless, single-earner married	0.0281
Two children, single	0.0235

Source: own calculation

as a percentage or grossing-up definitely and also other elements, approximating). However, due to the nature of the theoretical income tax system chosen by us, the tax model can closely approximate a system whose tax burden curves do not show major breakpoints (e.g., the termination of an allowance above a specified income without revocation[13]), or elements of relief that differentiate the way each type of family is taxed (e.g., Polish splitting or different use options within certain allowances[14]), only if these parameters are also applied in the theoretical system. At the same time, the reduction of the number of parameters used in the personal income tax systems of the examined countries should definitely be suggested: on the one hand due to the presented, substitutable or simplifiable factors, and, on the other hand, because even in the case of the most complex Polish tax system, the difference in tax payments calculated with the theoretical system did not exceed 3% of earnings-related income calculated in with the real system[15] (the same is also not much more, only 3.5 per cent, in the case of a modified theoretical system with the revocation of the child allowance). Let us also not forget that we have approximated the same theoretical model to the tax systems of all countries.

Naturally, given that the paper presented the opportunities of simplification on the basis of individual examples and not in general mathematical contexts, it is difficult to reach final, general conclusions, but we could still see our hypotheses confirmed based on the analysis. According to our first hypothesis, a given tax burden curve can also be defined using multiple basic taxation elements. We have confirmed this hypothesis with what has been written in connection with the application and substitutability of the tax credit and the tax-free bracket, and the detailed analyses of the paper also point in this direction. According to the second hypothesis, there is a specific system of personal income tax applied in practice, where the expected effect on the tax burden can be achieved with simpler means or may be approximated with good results. The functional approximations presented by the detailed analyses of the study in relation to the Czech, Slovak and Polish personal income tax systems also confirmed this hypothesis. For the study, we also used a new computer simulation system developed by the author, with the help of which the tax burden of a new tax system can be approximated to another and its parameters can be calculated.

The paper does not evaluate the tax principles that form the basis of tax burdens, and does not question their reasons. The aim of this paper is only to examine the approximation of tax burden functions created according to certain principles using simplified parameters. After all, the main goal of a decision-maker is to generate revenue through taxation and to influence the distribution of income in some way. The effect on revenue and income distribution is mathematically expressed with the tax burden curve, in relation to which, of course, it is important to understand which parameters will lead to its expected shape, but the most important aspect is the degree of the final burden after all. If we approach the nature of taxation from this point of view, the analysis suggests that the sophisticated fairness of the system and the use of the very complex allowance elements used in personal income tax systems should be carefully considered.

Appendix

CHARACTERISTICS OF THE TAX SYSTEMS OF THE EXAMINED COUNTRIES

Czech Republic

In 2018, the personal income tax base in the Czech Republic is super-gross, so the tax base also includes the contribution paid by the employer for social and health insurance (34 per cent of gross income). The personal income tax rate is 15 per cent, yet the employee also pays a 7 per cent special solidarity surcharge if his income exceeds the maximum amount of the social security contribution base (this is CZK 1,438,992 p.a. in 2018). However, the solidarity additional tax enters the system at a high income that is not included in the OECD data series examined.

In the Czech tax system, a tax credit is also available (CZK 24,840 per taxpayer), and the system also gives preference to low-income spouses (a tax allowance of CZK 24,840 is available for spouses with an annual income of less than CZK 68,000), and a tax allowance is also available for children, to a different extent according to the number of children - in the amount of CZK 15,204 p.a. for the first child and CZK 19,404 for the second child. In 2018, the system will also benefit children attending kindergarten: A tax allowance CZK 12,200 is available per pre-schooler.

Slovakia

In Slovakia, two tax rates were introduced in 2013, ranging from 19 per cent to 25 per cent. The annual upper limit of the 19 per cent tax bracket in 2018 was EUR 35,022.31, so the upper limit of the lower tax rate was set very high: income almost three times higher than the average income in 2018 was taxed only at the second tax rate. The second tax rate enters the system at a high income that is no longer covered by the OECD data series.

In Slovakia, the total social security and health insurance contributions of 13.4% payable on gross income are not included in the tax base, however, the amount of this tax base allowance is also affected by the health insurance contribution allowance: the health insurance contribution base must be reduced by EUR 4,560, which contribution allowance decreases with the increase in income (each EUR 1 monthly increase in income reduces the allowance by EUR 2) and then ceases at 57 per cent of the average earnings. The social security contribution on income above seven times the average wage in the second year before the tax year no longer increases, but this means such a high income that is no longer included in the OECD data series examined.

A personal allowance is also available in the tax base, amounting to EUR 3,830.02 below the income of EUR 19,948, decreasing above that income and then ceasing.

Of the family support elements, the spouse's tax base allowance is also decreasing parallel with the increase in income, the basic amount of which is EUR 3,830.02, but the amount also depends on the income of the taxpayer and the household income. The reduction in the allowance, depending on the taxpayer's income, is EUR 35,268.06, which means such a high income that it is no longer included in the OECD data series examined.

In the Slovak tax system, a tax allowance of EUR 21.56 per month and per child is available for children.

Poland

In Poland, a two-rate personal income tax system was in place in 2018: the lower rate was 18 per cent up to an income limit of PLN 85,528, and the portion above this income was taxed at 32 per cent.

Taxpayers living with a spouse can choose to file a joint tax return, adding up their income, or alone, so if one of them has a lower income, they can pay tax according to the lower tax bracket (splitting). This method of taxation is also available for by single taxpayers raising a child.

Social system contributions do not form the basis of personal income tax. The pension and disability contributions do not have to be paid on the part of the income above a certain income, this also affects the tax base allowance of the social system contributions and the tax allowance of the health insurance contribution. Employees can deduct at least PLN 1,335 from the tax base (to offset the work-related expenses), the amount of which may increase depending on the employee's job characteristics, such as commuting between their home and their place of work, whether they have a second job, etc.

A tax credit of PLN 556.02 can be deducted from the tax, which can be taken into account degressively from 2018 onwards as income increases. With regard to the health contribution paid by the employee (which is 9 per cent of gross income), 7.75 percentage points can be deducted from the tax, the allowance of which is reduced by contributions from the social system. The Polish tax system supports childbearing with a tax credit of PLN 1112.04 per child (deductible from tax), to which the possibility of a negative tax applies again from 2018.

Notes

[1] The statement that while the tax-free bracket is also favourable for higher income earners, the use of the tax credit already displaces earners above the credit income limit from this tax benefit (Ambrus, 2012) is true only with the same general tax rate and relatively high tax credit withdrawal rate.

[2] A system using a single tax rate and tax-free bracket or tax credit is referred to in the technical literature as a flat-rate tax system.

[3] Because of the limited scope of this paper, the program cannot be described in detail here, but the author will be happy to demonstrate the structure of the program upon request.

[4] http://stats.oecd.org, Public Sector, Taxation and Market Regulation / Taxation / Tax wedge decomposition

[5] In the Czech Republic, in addition to the child tax allowance, there is a separate tax allowance for pre-school children, and in Poland the amount of the allowance for work-related expenses varies depending on the employee's job characteristics: whether the employees commute between their place of residence and their place of work, have a second job, etc.

[6] See the Appendix for the characteristics of the Czech, Slovak and Polish personal income tax systems.

[7] The relative error (relative residual standard deviation) shows that the absolute error of the estimate is a fraction of the mean of the outcome variable (in this case, a root mean square was used in the calculations due to the frequent negative values). The absolute error (residual standard

deviation) expresses the average deviation of the approximated values from the observed values of the outcome variable.

[8] Given that with its outstanding deviation, it affects the total sum of squares too strongly.

[9] In relation to the allowance for pre-school children it must also be taken into account that in reality it does not apply to all taxpayers with children, but only to those raising a preschool child, however, as indicated in the description of the model, it was not possible to treat them as a separate taxpayer group.

[10] As above, with the exception of the contribution allowance.

[11] Due to the strong overlap of the graphs in pairs in the figure, the figure has been divided into two parts for better clarity.

[12] Here, too, the figure is divided into two parts.

[13] Tax allowance related to children in Poland.

[14] In the Czech Republic, the tax allowance for children can be considered as a negative tax, the tax allowance for pre-school children cannot.

[15] Not even at the income level with the largest differences, for childless single peoples, where the difference in tax payments between the theoretical and the real system is the largest.

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